Journal of Basic and Applied Engineering Research
p-ISSN: 2350-0077; e-ISSN: 2350-0255; Volume 6, Issue 7; July-September, 2019, pp. 385-389

http://www.krishisanskriti.org/Publication.html

An Alternative Method for Inference in Traffic Intensity

Kaustav Dutta[1] and Amit Choudhury[2]

[1]Research Scholar, Department of Statistics, Gauhati University
[2]Professor (on Lien), Department of Statistics, Gauhati University
E-mail: [1]kaustav0708@gmail.com, [2]achoudhury@rediffmail.com

Abstract—*The traffic intensity is perhaps the most widely used performance measure. We consider the problem of estimating traffic intensity of the M/M/1 queuing model. We propose an alternative method of estimating the traffic intensity of the M/M/1 queuing model. The advantage of our method lies in its simplicity. An approach for determination of sample size is also given.*

Keywords: *Brown- Forsythe Test; Consistency; Inference; M/M/1; Sufficiency; Unbiasedness.*

1. INTRODUCTION AND LITERATURE REVIEW:

Statistical inference in queuing theory is of recent origin. The aim is often to find the estimators of different performance measures of the queuing model. The literature of queuing theory outlines various performance measures of this model. The popular ones are average number of customers in the system (L), average number of customers in the queue (L_q), average waiting time in the system (W), average waiting time in the queue (W_q), the server utilization factor or traffic intensity (ρ). Typically, numerical analysis of queuing scenario involves computing these performance measures. In attempting to do so, a queuing analyst is often handicapped by the lack of widely accepted statistical procedures to estimate these performance measures.

There have been two broad approaches in statistical inference - the Classical and the Bayesian. These two approaches have their critics and fans. In the classical framework there have been a number of attempts. The pioneering work in this area is by Clarke (1957) who obtained maximum likelihood estimates for the arrival and service parameters of an M/M/1 queue. Lilliefors (1966) has considered the problem of finding confidence intervals for the actual M/M/1 traffic intensity from the maximum likelihood estimates given by Clarke. The Bayesian approach in the field of inference in queuing theory largely owes its development to the work by Armeroand Bayarri (1994a, 1994b, 1999, 2000). Sharma and Kumar (1999) discussed statistical inference both from frequentist as well as from Bayesian perspective. Choudhury and Borthakur (2008) derived Bayes estimators of performance measures with respect to squared error loss function. Zheng and Seila (2000) have mathematically established the nonexistence of expectations and standard errors of common estimators of performance measures. Chowdhury and Mukherjee (2011) carried out the estimation of waiting time in the M/M/1 model in the form of its right tail area known as exceedance probability. Srinivas et al. (2011)discussed uniform minimum variance unbiased estimators (UMVUE) and maximum likelihood estimators (MLE) and suggested the use of UMVU estimators over ML estimators for some measures and Chowdhury and Mukherjee (2013) constructed the MLE and Bayes estimator of ρ considering the M/M/1 queuing model. Srinivas and Udupa (2014) developed best unbiased estimation and CAN property for performance measures in a stable M/M/1 Queue depending on system size at departure epochs. Srinivas and Kale (2016) obtained ML and UMVU estimation of performance measures in the M/D/1 queuing system. Recently, under Bayesian framework, Cruz et al. (2017) discussed Bayesian estimation in M/M/S model by generating data at departure epoch. They used beta distribution as prior for ρ. Almeida and Cruz (2017) studied M/M/1 queuing model by considering the number of customers left behind in the system at departure epochs. They used Jeffreys prior to obtain the posterior distributions of some parameters of interest and compared Choudhury and Borthakur (2008) priors Beta and Truncated Uniform prior via simulation with the Jeffreys prior and concluded that all the three priors could be considered acceptable.

Very recently, Suyama et al. (2018) considered Markovian multi server queues where they derived MLE of ρ in the M/M/s queues and showed that the estimate is equivalent to the moment estimator. Choudhury and Basak (2018) considered the M/M/1 model and constructed MLE of ρ and discussed its theoretical properties – consistency, complete sufficiency and unbiasedness. Cruz et al. (2018) considered M/M/1/k queuing model where they extended results for infinite Markovian queues to finite Markovian queues.

This paper is structured as follows. Section 2 contains the description of M/M/1 queue. Section 3 provides the procedure of estimating the traffic intensity. Section 4 contains sample size determination and theoretical framework is given in section 5. Section 6 contains our analysis. Section 7 provides our discussion. Section 8 concludes our paper.

2. THE M/M/1 QUEUING SYSTEM:

The single server Markovian queue is a queuing system with the following assumptions.

I. The time between the successive arrival of the customers is random and follows $\exp(\lambda)$.
II. The time required to serve each customer by the server i.e. the service time distribution is also random and follows $\exp(\mu)$.
III. There is only one server.
IV. System Capacity is infinite.
V. The customers are served on First Come First Serve basis (FCFS).
VI. Calling population is infinite.

Any queuing analyst has the general interest to evaluate the performance measures of this model. These measures are (Gross and Harris,2008)

I. Mean system size (L_s), $L = \lambda/(\mu - \lambda)$
II. Mean queue size (L_q), $L_q=\lambda^2/(\mu(\mu - \lambda))$
III. Average waiting time in system (W_s), $W = 1/(\mu - \lambda)$
IV. Average waiting time in queue (W_q), $W_q = \lambda/(\mu(\mu - \lambda))$
V. Traffic intensity (ρ), $\rho = \lambda/\mu$ (In order to ensure that the queue size does not explode, it is necessary that $\rho < 1$)

3. ESTIMATION OF TRAFFIC INTENSITY

Let $x_1,x_2,\ldots,x_{n_1}$ be independent and identically distributed random samples of size n_1 from inter arrival time whose distribution is exponential with parameter λ and let $y_1,y_2,\ldots,y_{n_2}$ be independent and identically distributed random samples of size n_2 from service time whose distribution is exponential with parameter μ. It is also assumed that $x_1,x_2,\ldots,x_{n_1}$ and $y_1,y_2,\ldots,y_{n_2}$ are mutually independent.

Now, further define $z_1 = \sum_{i=1}^{n_1} x_i \sim \gamma\,(n_1,\lambda)$ and $z_2 = \sum_{j=1}^{n_2} y_j \sim \gamma\,(n_2,\mu)$. Then, the distribution of z_1 and z_2, respectively is given by (Rohatgi &Ehsanes Saleh ,2001)

$$f(z_1) = \frac{\lambda^{n_1}}{\Gamma(n_1)} e^{-\lambda z_1} {z_1}^{n_1-1}, z_1 > 0$$

$$f(z_2) = \frac{\mu^{n_2}}{\Gamma(n_2)} e^{-\mu z_2} {z_2}^{n_2-1}, z_2 > 0$$

We have the joint distribution of z_1 and z_2 is

$$f(z_1 z_2) = \frac{\lambda^{n_1}}{\Gamma(n_1)} e^{-\lambda z_1} {z_1}^{n_1-1} \frac{\mu^{n_2}}{\Gamma(n_2)} e^{-\mu z_2} {z_2}^{n_2-1}, \quad z_1 > 0, z_2 > 0$$

$$= \frac{\lambda^{n_1}}{\Gamma(n_1)} \frac{\mu^{n_2}}{\Gamma(n_2)} e^{-\lambda z_1 - \mu z_2}\, {z_1}^{n_1-1} {z_2}^{n_2-1} (1)$$

We know make the following transformation

$$u_3 = \frac{z_2}{z_1} \text{ and } u_4 = z_1 (2)$$

The Jacobian of transformation is

$$|J| = \begin{vmatrix} \frac{dz_1}{du_3} & \frac{dz_2}{du_3} \\ \frac{dz_1}{du_4} & \frac{dz_2}{du_4} \end{vmatrix} = \begin{vmatrix} 0 & u_4 \\ 1 & u_3 \end{vmatrix} = u_4$$

We have from (1)

$$f(u_3 u_4) = \frac{\mu^{n_2}}{\Gamma(n_2)} e^{-\lambda u_4 - \mu u_3 u_4} {u_4}^{n_1-1} (u_3 u_4)^{n_2-1} u_4, \quad u_3 > 0, u_4 > 0$$

$$= \frac{\lambda^{n_1}}{\Gamma(n_1)} \frac{\mu^{n_2}}{\Gamma(n_2)} e^{-\lambda u_4 - \mu u_3 u_4} {u_4}^{n_1+n_2-1} {u_3}^{n_2-1}$$

$$\therefore f(u_3) = \int_0^\infty f(u_3 u_4) du_4$$

$$= \int_0^\infty \frac{\lambda^{n_1}}{\Gamma(n_1)} \frac{\mu^{n_2}}{\Gamma(n_2)} e^{-\lambda u_4 - \mu u_3 u_4} {u_4}^{n_1+n_2-1} {u_3}^{n_2-1}\, du_4$$

$$= \frac{\lambda^{n_1}}{\Gamma(n_1)} \frac{\mu^{n_2}}{\Gamma(n_2)} {u_3}^{n_2-1} \frac{\Gamma(n_1+n_2)}{(\lambda+\mu u_3)^{n_1+n_2}}$$

$$\therefore f(u_3) = \frac{\lambda^{n_1}}{\Gamma(n_1)} \frac{\mu^{n_2}}{\Gamma(n_2)} {u_3}^{n_2-1} \frac{\Gamma(n_1+n_2)}{(\lambda+\mu u_3)^{n_1+n_2}}\ ,\ u_3 > 0, (3)$$

We have,

$$E(u_3) = \int_0^\infty u_3 f(u_3) du_3$$

$$= \int_0^\infty \frac{\lambda^{n_1}}{\Gamma(n_1)} \frac{\mu^{n_2}}{\Gamma(n_2)} {u_3}^{n_2} \frac{\Gamma(n_1+n_2)}{(\lambda+\mu u_3)^{n_1+n_2}} du_3$$

$$= \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)} B(n_1-1, n_2+1) \frac{\lambda}{\mu}$$

$$= \frac{n_2}{n_1-1} \frac{\lambda}{\mu}$$

$$\therefore E\left(\frac{n_1-1}{n_2} u_3\right) = \frac{\lambda}{\mu} (4)$$

Hence, $\frac{n_1-1}{n_2} u_3$ is an unbiased estimator of ρ

Again,

$$E\left(\frac{n_1-1}{n_2} u_3\right)^2 = \int_0^\infty \left(\frac{n_1-1}{n_2} u_3\right)^2 f(u_3) du_3$$

$$= \left(\frac{n_1-1}{n_2}\right)^2 \int_0^\infty \frac{\lambda^{n_1}}{\Gamma(n_1)} \frac{\mu^{n_2}}{\Gamma(n_2)} {u_3}^{n_2+2-1} \frac{\Gamma(n_1+n_2)}{(\lambda+\mu u_3)^{n_1+n_2}} du_3$$

$$= \left(\frac{n_1-1}{n_2}\right)^2 \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)} B(n_1-2, n_2+2) \left(\frac{\lambda}{\mu}\right)^2$$

$$= \frac{(n_2+1)(n_1-1)}{n_2(n_1-2)} \left(\frac{\lambda}{\mu}\right)^2 (5)$$

$$\therefore V\left(\frac{n_1-1}{n_2} u_3\right) = \frac{(n_1+n_2-1)}{n_2(n_1-2)} \left(\frac{\lambda}{\mu}\right)^2 \text{ (using (4) and (5))}$$

So, $V\left(\frac{n_1-1}{n_2} u_3\right) \to 0$ as $n_1 \to \infty, n_2 \to \infty$

Hence, $\frac{n_1-1}{n_2} u_3$ is a consistent estimator of ρ.

Again from (3),

$$f(u_3) = \frac{\lambda^{n_1}}{\Gamma(n_1)} \frac{\mu^{n_2}}{\Gamma(n_2)} {u_3}^{n_2-1} \frac{\Gamma(n_1+n_2)}{(\lambda+\mu u_3)^{n_1+n_2}},$$

$u_3 > 0$

$$= \left(\frac{1}{\rho}\right)^{n_2} \frac{1}{\left(1+\frac{u_3}{\rho}\right)^{n_1+n_2}} \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)} {u_3}^{n_2-1}$$

$=g(t,\rho)h(u_3)$ where $t = u_3$

Hence, by using Neymann Factorization Theorem, u_3 is a sufficient estimator of ρ.

i.e. $\frac{n_1-1}{n_2} u_3$ is a sufficient estimator of ρ.

4. DETERMINATION OF SAMPLE SIZE

We generate a sample of inter arrival times using simulation technique under the Markovian setup with parameter λ. Similarly, we shall generate a sample from service time using simulation technique under Markovian setup with parameter μ. The simulation technique used to carry out our study is the method of inverse transformation. The algorithm is discussed by H.A. TAHA (1988)

We simulate random samples of inter-arrival times and service times under Markovian setup with different combination of λ and μ. For each of the pairs of λ and μ we obtain estimates of traffic intensity using various combination of sample sizes for inter arrival times of the customers (n_1) and service time of the customers (n_2) respectively. Using the above procedure, we simulate 8000 estimates of the traffic intensity for each pair of sample sizes.

5. THEORETICAL FRAMEWORK

Now regarding the estimates of traffic intensity, Zheng and Seila (2000) prescribed " If, after collecting samples of interarrival times and service times, the sample traffic intensity, $\hat{\rho}$, larger than ρ_0 (whereρ_0 is a known constant), the $\rho_0$would be used in place $\hat{\rho}$ in the substitution estimators".

Regarding the value ρ_0, Dutta and Choudhury (in press) prescribed that "In practice, an analyst can generally specify an upper bound on the acceptable level of system congestion, i.e., an upper bound on the acceptable values of ρ. Call this upper bound ρ^*. Then, ρ_0 should be so chosen that it is as close to ρ^* as possible (in the range $\rho^* < \rho_0 < 1$)".

6. ANALYSIS

For analysis we considered the following typical choices of λ and μ.

1. λ= 12, μ = 15
2. λ= 10, μ = 16
3. λ= 16, μ = 18

we consider following possible choices of ρ_0.

For case 1 ρ_0= 0.82, 0.85

For case 2. ρ_0= 0.65,0.67

For case 3. ρ_0= 0.91,0.93

For the typical choice of λ and μ as stated above, we generate random samples of inter arrival times and service times for the following sample sizes viz.

n_1= (25,50,75,100,125) and

n_2= (25,50,75,100,125)

Now, we perform homogeneity of variance test between two pairs of n_1 and n_2. If for any two pair, the test shows non-significant results. Then we will accept the hypothesis of equality of variance. There will be no need of increasing the sample size after that. And the pair of n_1 and n_2 could be considered as the required sample size.

We shall perform Brown- Forsythe test for homogeneity of variance. The reason behind it is that it is more robust and is relatively insensitive to departures from normality (Brown and Forsythe, 1974).

We place below the results for the first case due to constraint of space.

Table 1: Brown Forsythe Test for λ= 12, μ = 15 with n_1 and n_2 pairs (25,100), (25,125) and ρ_0= 0.82

Brown Forsythe Test					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.008	1	.008	1.771	.183
Within Groups	75.526	15998	.005		
Total	75.534	15999			

Table 2: Brown Forsythe Test for λ= 12, μ = 15 with n_1 and n_2 pairs (100,25), (125,25) and ρ_0= 0.82

Brown Forsythe Test					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.001	1	.001	.186	.666
Within Groups	93.836	15998	.006		
Total	93.837	15999			

Table 3: Brown Forsythe Test for λ= 12, μ = 15 with n_1 and n_2 pairs (25,100), (25,125) and ρ_0= 0.85

Brown Forsythe Test					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.016	1	.016	3.569	.059
Within Groups	69.488	15998	.004		
Total	69.504	15999			

Table 4: Brown Forsythe Test for λ= 12, μ = 15 with n_1 and n_2 pairs (100,25), (125,25) and ρ_0= 0.85

Brown Forsythe Test					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	.000	1	.000	.087	.768
Within Groups	85.342	15998	.005		
Total	85.343	15999			

7. DISCUSSION

Table 1 gives the Brown Forsythe Test for λ= 12, μ = 15 with n_1 and n_2 pairs (25,100), (25,125) and ρ_0= 0.82. we observe that p value = 0.183> 0.05. so, we can accept our null hypothesis of homogeneity of variance. There will be no need of increasing the sample size after that.

Table 2 gives the Brown Forsythe Test for λ= 12, μ = 15 with n_1 and n_2 pairs (100,25), (125,25) and ρ_0= 0.82. we observe that p value = 0.666> 0.05. so, we can accept our null hypothesis of homogeneity of variance. There will be no need of increasing the sample size after that.

Table 3 gives the Brown Forsythe Test for λ= 12, μ = 15 with n_1 and n_2 pairs (25,100), (25,125) and ρ_0= 0.85. we observe that p value = 0.059 > 0.05. so, we can accept our null hypothesis of homogeneity of variance. There will be no need of increasing the sample size after that.

Table 4 gives the Brown Forsythe Test for λ= 12, μ = 15 with n_1 and n_2 pairs (100,25), (125,25) and ρ_0= 0.85. we observe that p value = 0.768> 0.05. so, we can accept our null hypothesis of homogeneity of variance. There will be no need of increasing the sample size after that.

8. CONCLUSION

From above discussion we can observe that the hypothesis of homogeneity of variance can be accepted for a total sample of size 125 (approx.). We therefore prescribe that a minimum total sample size of 125 may be used in such a manner that one of the components (n_1 or n_2) is at least 25.

A caveat is in order. The conclusion that we have drawn is on the basis of simulation. We invite other researchers to carry out similar exercises and confirm or improve upon our conclusion.

REFERENCES

[1] Almeida M. A. C and Cruz F. R. B (2017): A note on Bayesian estimation of traffic intensity in single-server markovian queues, Communications in Statistics -Simulation and Computation, DOI: 10.1080/03610918.2017.1353614.

[2] Armero C, Bayarri MJ (1994a). Bayesian prediction in M/M/1 queues. Queuing Syst 15:401-417.

[3] Armero C, Bayarri MJ (1994b). Prior assessments for prediction in queues. Statistician ,43(1):139-153.

[4] Armero C, Bayarri MJ (1999). Dealing with uncertainties in queues and networks of queues: a Bayesian approach. In: Ghosh S (ed) Multivariate analysis, design of experiments and survey sampling. Marcel Dekker, New York, pp 579-608.

[5] Armero C, Bayarri MJ (2000). Queues. ISDS Discussion paper No 39.

[6] Brown M. B, Forsythe A.B (1974). Robust tests for the equality of variances. Journal of the American Statistical Association, Vol. 69, No. 346, pp. 364-367.

[7] Choudhury A, Borthakur, A. C. (2008). Bayesian inference and prediction in the single server Markovian queue. Metrika 67:371-383.

[8] Choudhury A, Basak A (2018). Statistical inference on traffic intensity in an M/M/1 queueing system, International Journal of Management Science and Engineering Management, DOI:10.1080/17509653.2018.1436010.

[9] Chowdhury S, Mukherjee S.P. (2011). Estimation of waiting time distribution in an M/M/1 Queue. OPSEARCH (Oct-Dec 2011) 48(4): 306-317.

[10] Chowdhury S, Mukherjee S.P. (2013). Estimation of traffic intensity based on queue length in a single M/M/1 queue, Communications in Statistics- Theory and Methods, 42:13,2376-2390.

[11] Clarke A.B. (1957). Maximum likelihood estimates in a simple queue. Ann Math Stat 28:1036-1040.

[12] Cruz F.R.B, Quinino R.C. and Ho L.L (2017). Bayesian estimation of traffic intensity based on queue length in a multi-server M/M/S queue. Communication in Statistics- Simulation and Computation,46(9), 7319-7331.

[13] Cruz F. R. B, Almeida M. A. C, D'Angelo M.F.S.V and Woensel T. V (2018). Traffic intensity estimation in finite Markovian queueing systems. Hindawi, Mathematical Problems in Engineering, volume 2018, https://doi.org/10.1155/2018/3018758.

[14] Dutta K and Choudhury A (*in Press*). Estimation of Performance Measures of M/M/1 Queues- A Simulation based Approach. *International Journal of Applied Management Science.* (Journal Forthcoming Issue)

[15] Gross D. and Harris C.M. (2008). Fundamentals of queuing theory, 4^{th}edn, Wiley.

[16] Lilliefors HW (1966). Some confidence intervals for queues. Oper Res 14, 723–727.

[17] Rohatgi V. K, Ehsanes Saleh A. K. Md (2001). An introduction to probability and statistics. Wiley series in probability and statistics. 2nded., New York: Wiley. DOI: 10.1002/9781118165676

[18] Sharma K.K, Kumar V (1999). Inference on M/M/1:(∞:FIFO) queue systems. OPSEARCH 36(1):26-34.

[19] Srinivas V, Rao S.S, Kale B.K (2011). Estimation of measures in M/M/1 queue, Communications in Statistics- Theory and Methods, 40:18, 3327-3336.

[20] Srinivas V, Jayakrishna Udupa H (2014). Best unbiased estimation and CAN property in the stable M/M/1 queue, Communications in Statistics- Theory and Methods, 43:2, 321-327.

[21] Srinivas V and Kale B. K (2016). ML and UMVU estimation in the M/D/1 queuing system. Communications in Statistics – Theory and Methods, 45(19), 5826-5834.

[22] Suyama E, Quinino R. C and Cruz F.R. B (2018). Simple and yet efficient estimators for Markovian multiserver queues. Hindawi, Mathematical Problems in Engineering, volume 2018, https://doi.org/10.1155/2018/3280846.

[23] Taha HA (1988). Simulation modelling and SIMNET, Prentice – Hall, Englewood Cliffs, New Jersey.

[24] Zheng S, Seila AF (2000). Some well-behaved estimators for the M/M/1 queue. Oper Res Lett 26(5):231-235.